Lesson 8

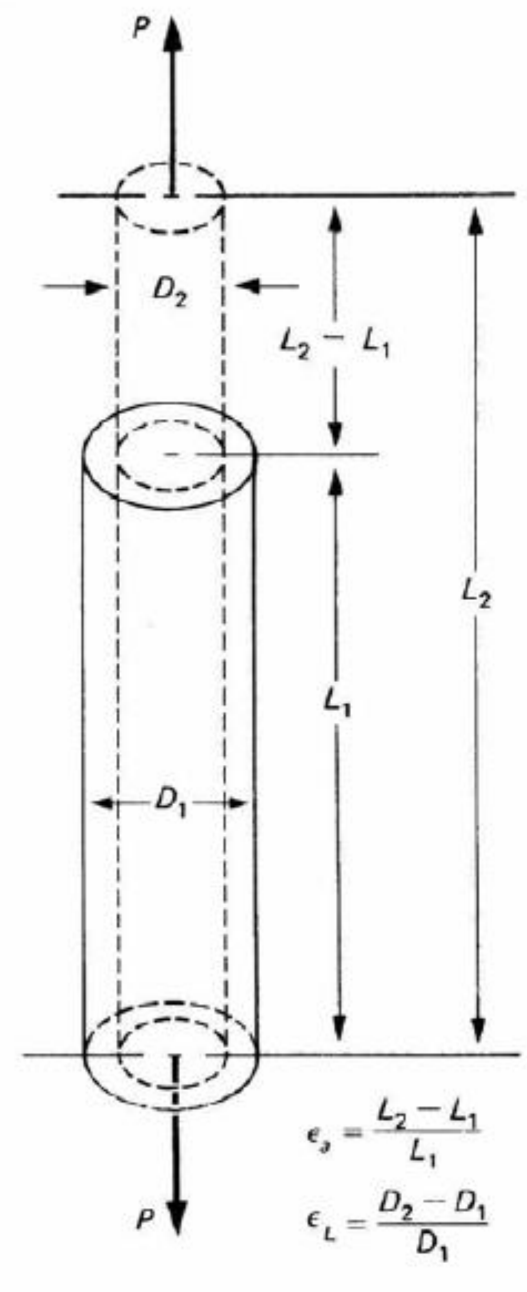
Thermomechanical Measurements for Energy Systems (MENR)

Measurements for Mechanical Systems and Production (MMER)

A.Y. 2015-16

Zaccaria (Rino) Del Prete

Measurement of Mechanical STRAIN



Strain measurements are perhaps the most widespread measurements done in engineering (*tension, force, pressure, …*):

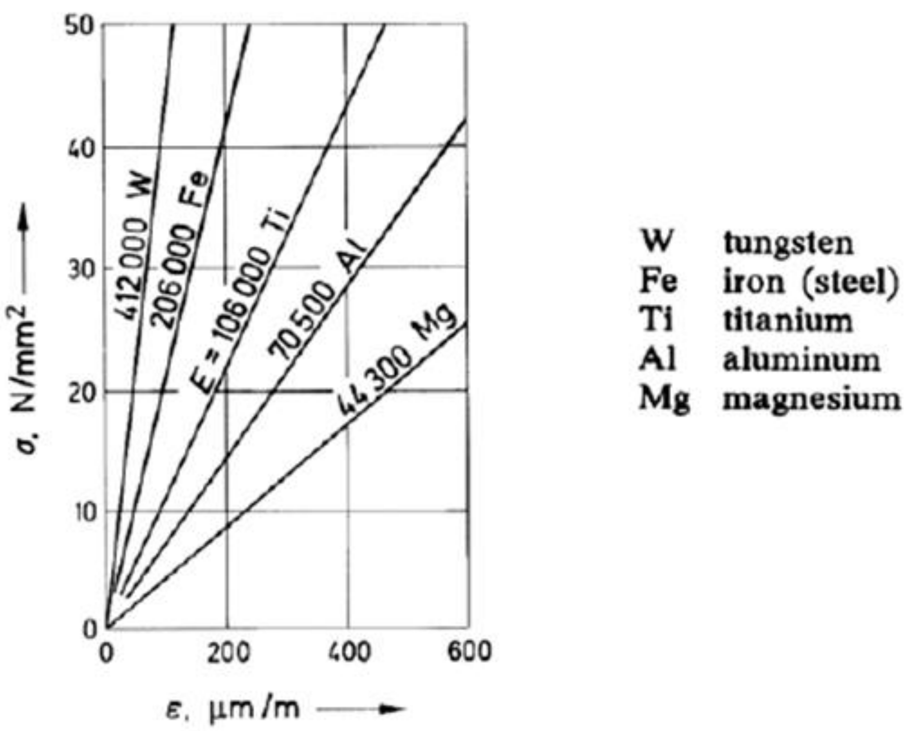
$$\varepsilon_a = \frac{L_2 - L_1}{L_1} = \frac{\Delta L}{L}$$

We measure here the *lengthening and shortening* of mechanical structures. If we know the ***Young's Modulus E*** of the material, after measuring the strain ε_a we can obtain the value of ***mechanical tension σ = E·ε*** *!*

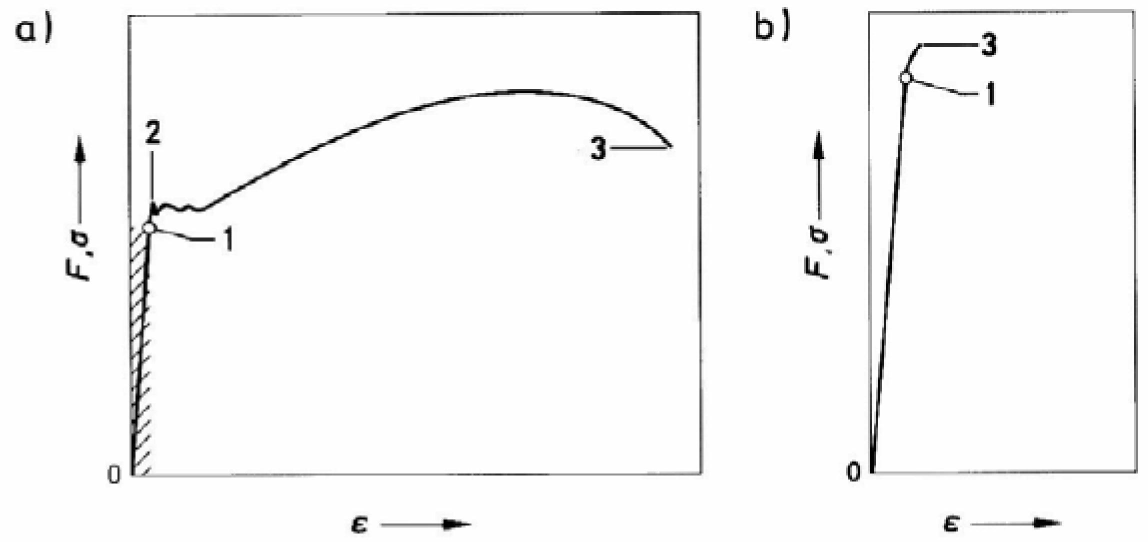
Transverse strain is measured by considering the *radial contraction* $(D_2 - D_1)/D_1$ and involves the knowledge of the

Poisson Coefficient: $\nu = \frac{\varepsilon_t}{\varepsilon_a}$

Which for metallic materials ranges from 2 up to 4 …
Conventionally ***ν = 0,3***



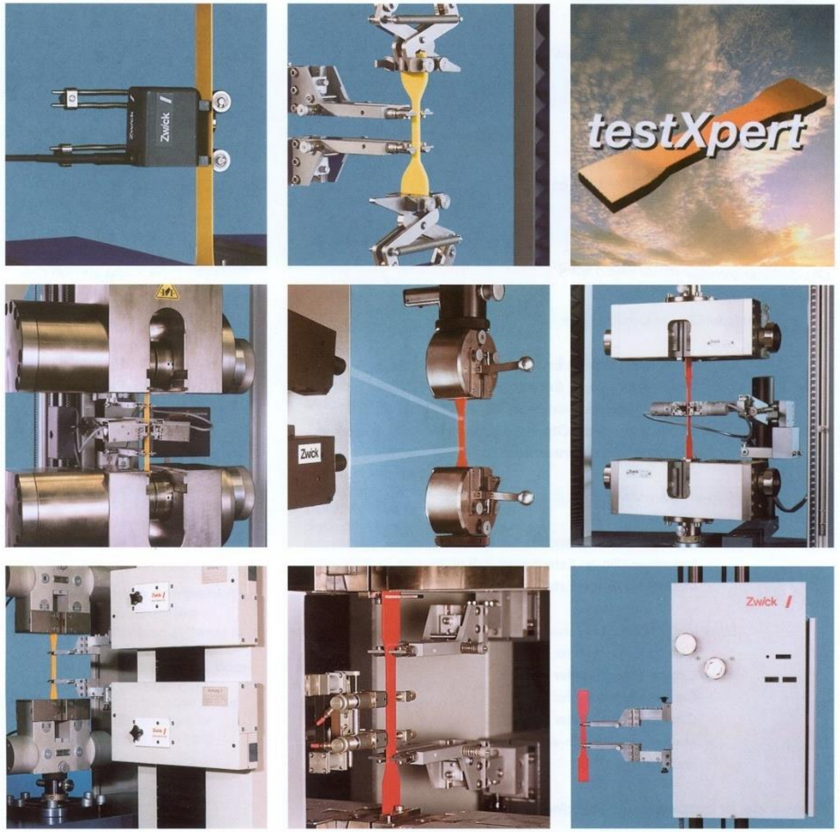
***Ductile* and *brittle* materials:**



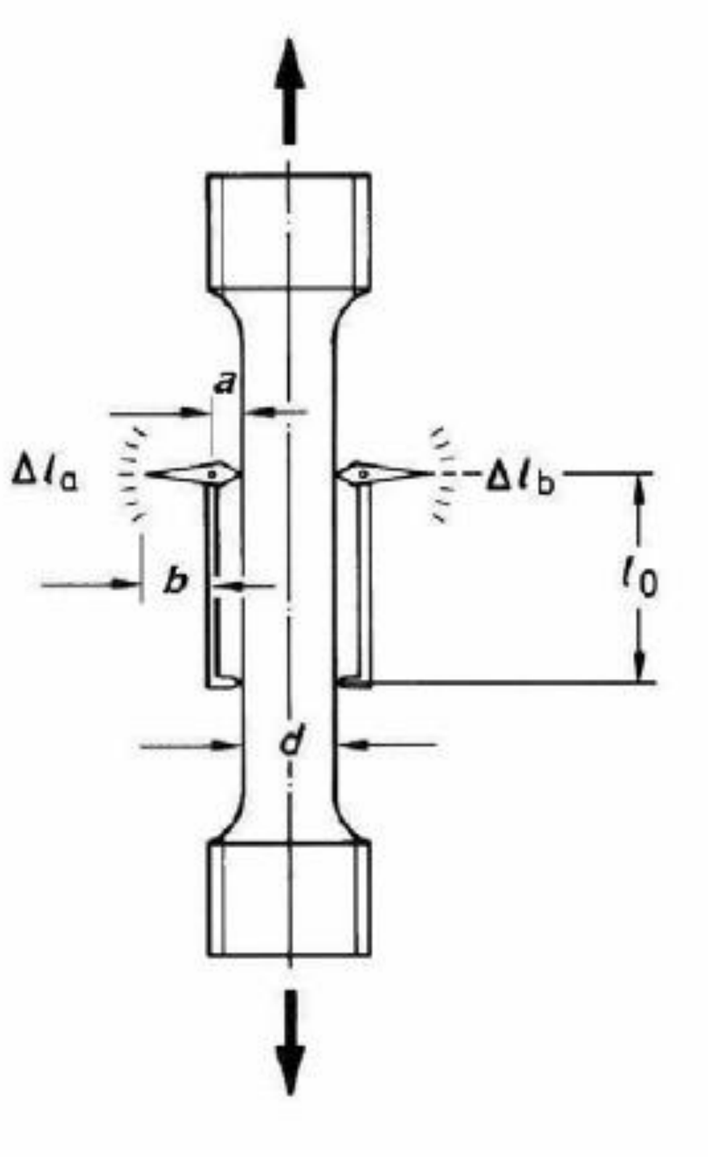
1. limit of elasticity,
2. yield point,
3. fracture.

Examples of force/strain and stress/strain curves.
a) Characteristic of a ductile material with large yield range
b) Characteristic of a brittle material

A first type of instruments used to measure the *strain* or the *displacement* between two adjacent points are the ***Mechanical Extensometers …***



Mechanical Extensometers :



The displacement is measured between two points, initially at distance l_0

When the specimen elongates Δl, the indicator rotates of an angle ϑ and we reed the output λ …

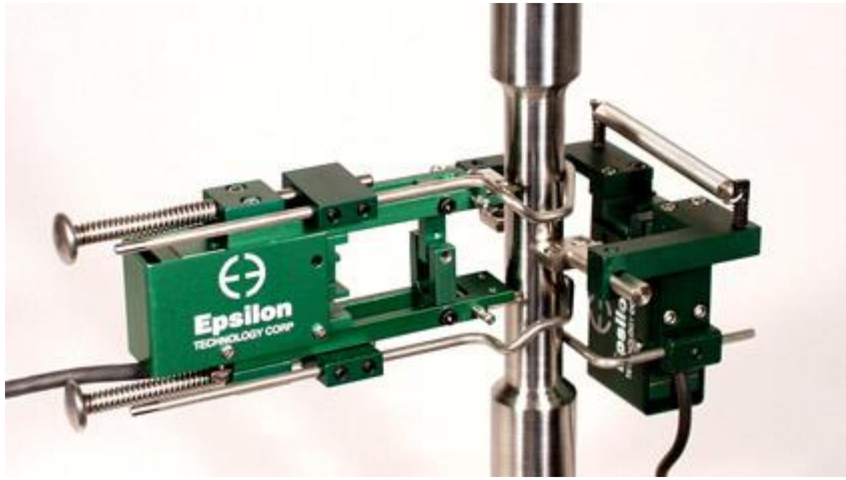
$$\Delta l = a \cdot \theta$$

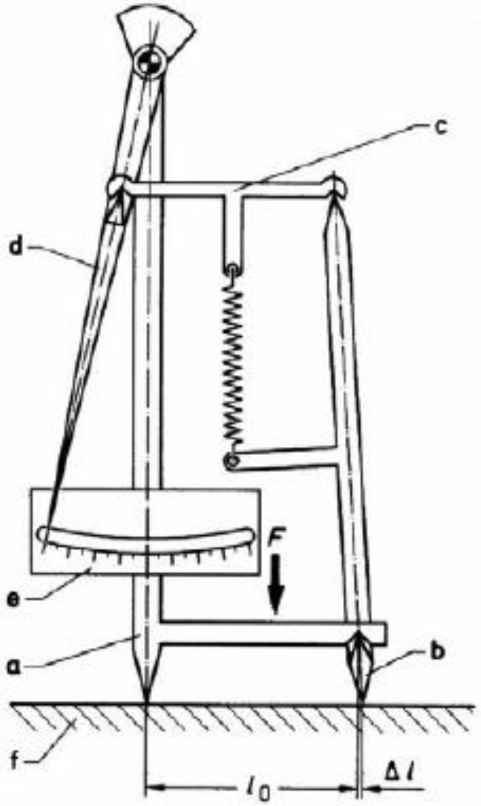
$$\lambda = b \cdot \theta$$

The ratio is : $\dfrac{\Delta l}{\lambda} = \dfrac{a}{b}$

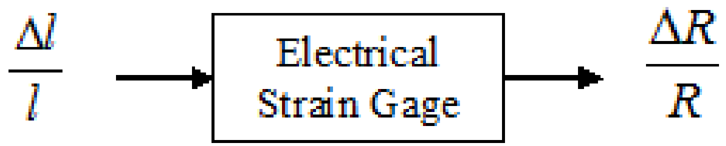
And the graduation curve is then:

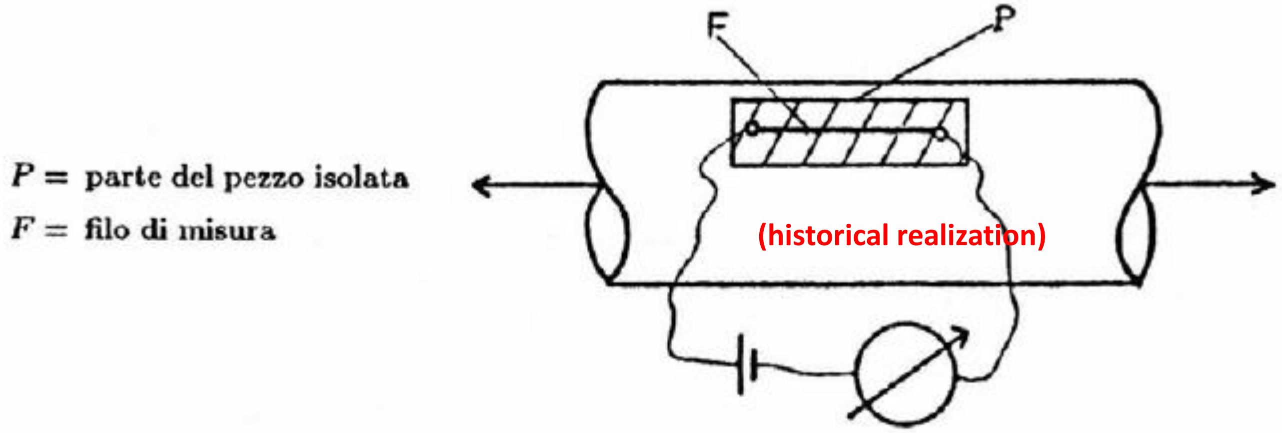
$$\varepsilon = \frac{\Delta l}{l_0} = \frac{a\lambda}{l_0 b}$$



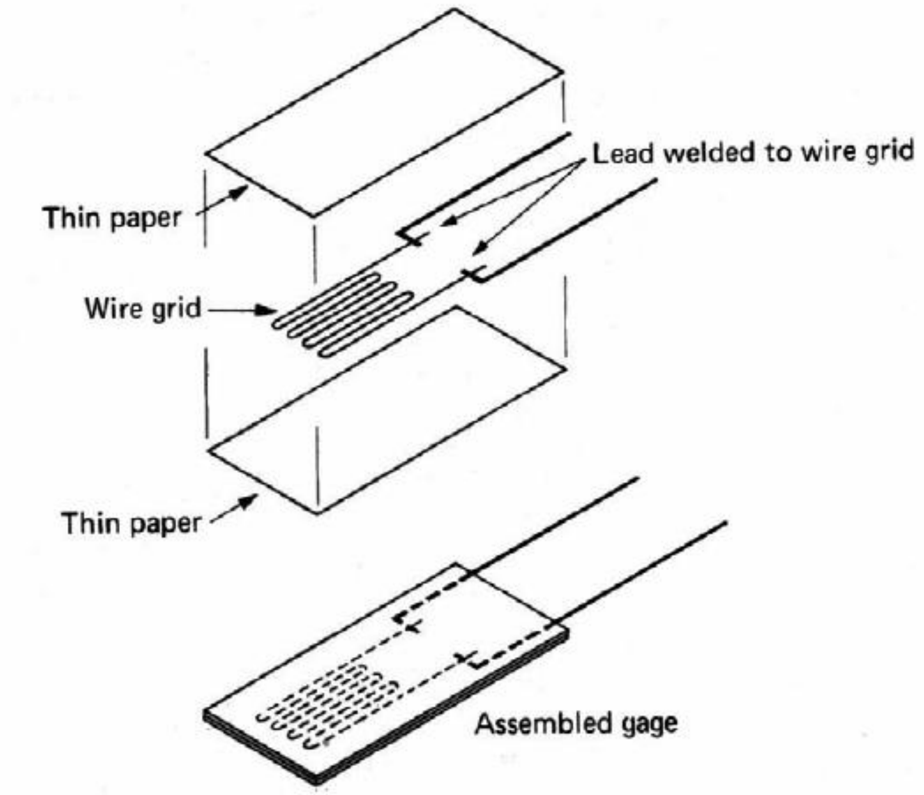


Electrical STRAIN GAGES :





(historical realization)



(modern realization)

However, what we really need is a *punctual strain measurement*, so to effectively monitor the zones of the surface where the strain concentrates.

Electrical strain gages are based on resistance variation with elongation !

$$R = \rho \frac{l}{S}$$

First we have to elaborate a bit the basic physical relation of ***resistivity*** …

$$\ln R = \ln\left(\rho \frac{l}{S}\right) = \ln \rho + \ln l - \ln S$$

then we derivate the relation and we transform the *infinitesimal terms* into ***finite differences …***

$$\frac{1}{R}\frac{dR}{dt}=\frac{1}{\rho}\frac{d\rho}{dt}+\frac{1}{l}\frac{dl}{dt}-\frac{1}{S}\frac{dS}{dt}$$

$$\frac{\Delta R}{R}=\frac{\Delta\rho}{\rho}+\frac{\Delta l}{l}-\frac{\Delta S}{S}$$

Because it is $S=\pi\frac{D^2}{4}$ and $\frac{dS}{dt}=\frac{\pi}{4}2D\cdot\frac{dD}{dt}\rightarrow\frac{\Delta S}{\Delta t}=\frac{\pi}{2}D\cdot\frac{\Delta D}{\Delta t}\rightarrow\Delta S=\frac{\pi}{2}D\cdot\Delta D$

$$\frac{\Delta S}{S}=\frac{\frac{\pi}{2}\cdot D\cdot\Delta D}{\frac{\pi}{4}\cdot D^2}=2\frac{\Delta D}{D}=2\cdot\varepsilon_t$$

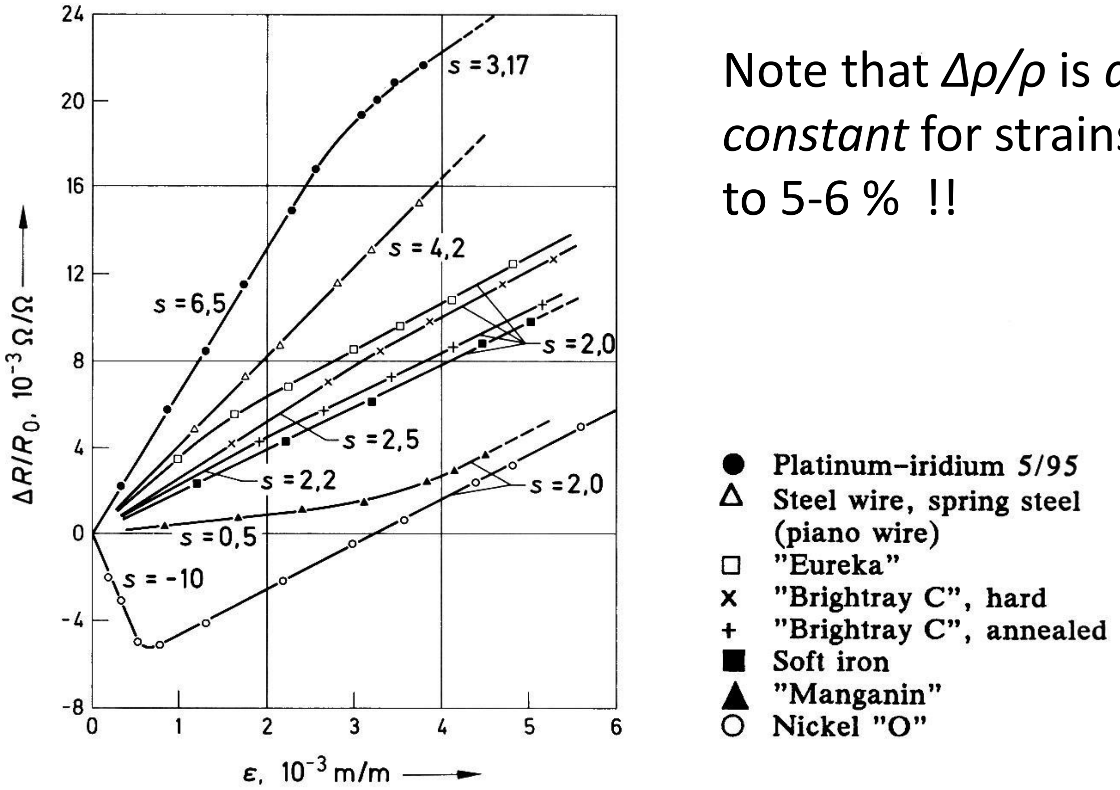
it remains $\frac{\Delta R}{R}=\frac{\Delta\rho}{\rho}+\frac{\Delta l}{l}-2\frac{\Delta D}{D}=\frac{\Delta\rho}{\rho}+\varepsilon_a-2\varepsilon_t=\frac{\Delta\rho}{\rho}+\varepsilon_a+2\cdot\nu\varepsilon_a$ (ε_t sign is the opposite of ε_a)

$$F=\frac{\frac{\Delta R}{R}}{\frac{\Delta l}{l}}=\frac{\frac{\Delta\rho}{\rho}}{\frac{\Delta l}{l}}+1+2\nu$$

with $\nu\cong 0.3$ we would have $F\cong 1.5\div 1.7$
however, variation of *Δρ/ρ* is <u>not zero</u> !

Because the contribution of the *resistivity variation Δρ* is difficult to calculate, strain gage producer *proceed experimentally* and «measure» the *factor F* for a certain number of gages.
The resulting number is then assigned to the same lot of transducer … that is the ***gage factor*** $F \simeq 2$

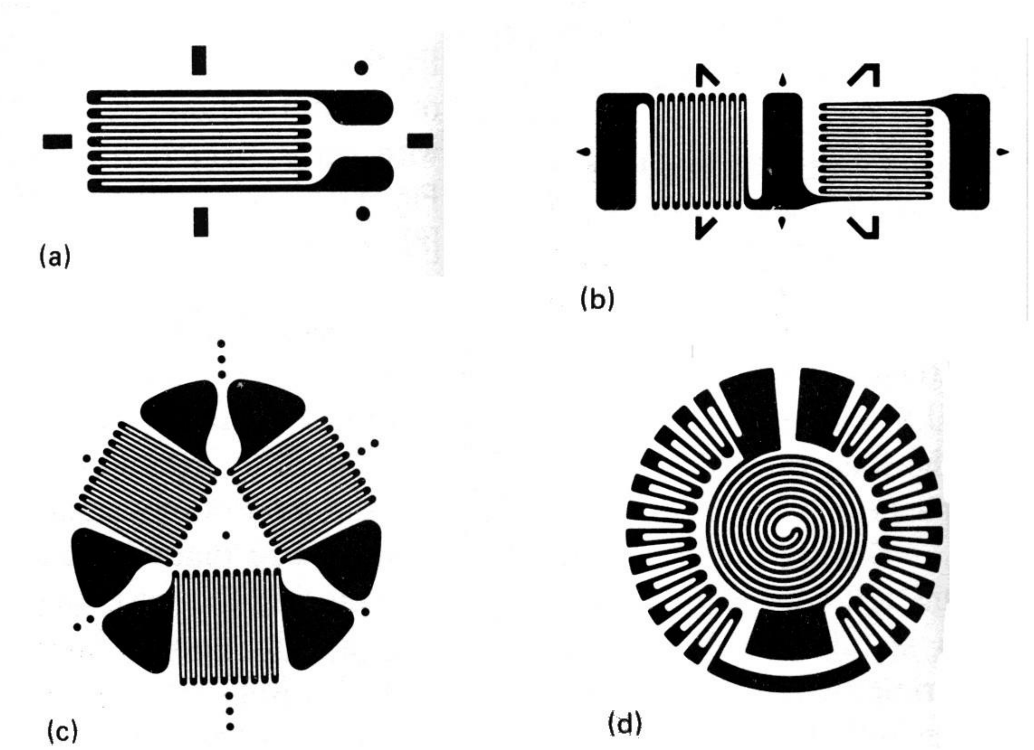
Strain gage «graduation curve» $$\frac{\Delta R}{R} = F \cdot \varepsilon$$



Resistance/strain characteristics for freely tensioned wires

Note that *Δρ/ρ* is *almost constant* for strains up to 5-6 % !!

Commercial strain gages :



Thermal effects :

Unfortunately, because an electric current is flowing through every strain gage, there is an additional ***thermal contribution*** on resistance variation: $\frac{\Delta R'}{R} = \alpha \cdot \Delta T$

Due to heating, the strain gage wire undergoes also a *thermal elongation* $\frac{\Delta l'}{l} = \beta' \cdot \Delta T$ which, moreover, is generally NOT equal to the *thermal elongation of the underlying material* $\frac{\Delta l''}{l} = \beta'' \cdot \Delta T$ We have therefore:

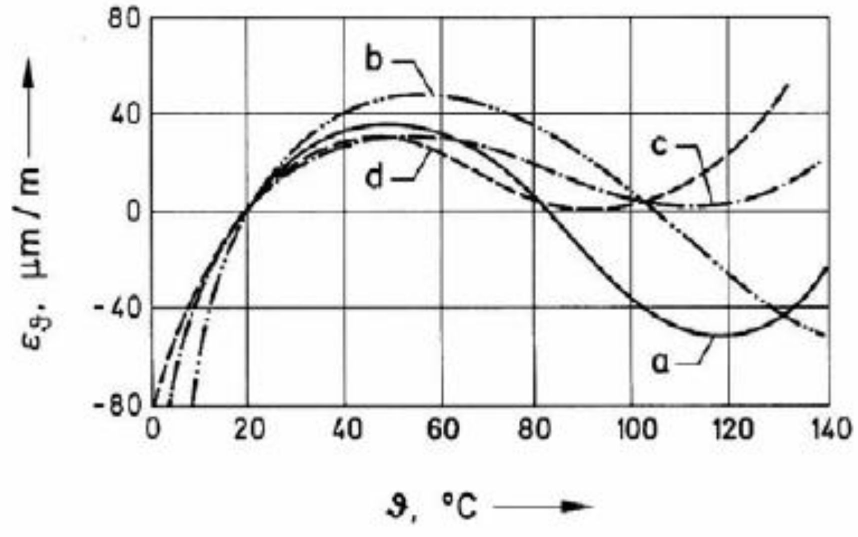
1. a resistance variation due to ***mechanical strain***: $\frac{\Delta R}{R} = F \cdot \varepsilon$
2. a resistance variation due ***only to thermal effects***: $\frac{\Delta R_T}{R} = \left[\alpha + F(\beta' - \beta'')\right] \cdot \Delta T$

There is always an ***"apparent strain"*** which, in fact, is NOT real : $\varepsilon_{app} = \frac{1}{F}\frac{\Delta R_T}{R} = \left[\frac{\alpha}{F} + (\beta' - \beta'')\right] \cdot \Delta T$

Because the coefficients α and β' depend on the gage wire material while the coefficient β'' depends on the underlying material, if we can make $\alpha + F(\beta' - \beta'') = 0$ then we are nulling the apparent thermal contribution ε_{app}

Temperature self-compensated strain gages approximately realize: $\frac{\alpha}{F} = \beta'' - \beta'$

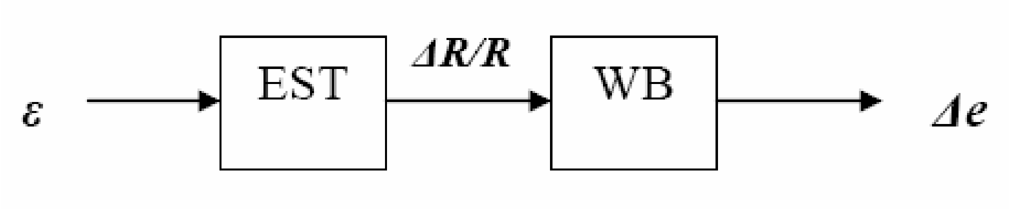
Temperature self-compensation is not perfect, nor constant with temperature, and it works ONLY if the strain gage is applied on the "correct underlying material" …

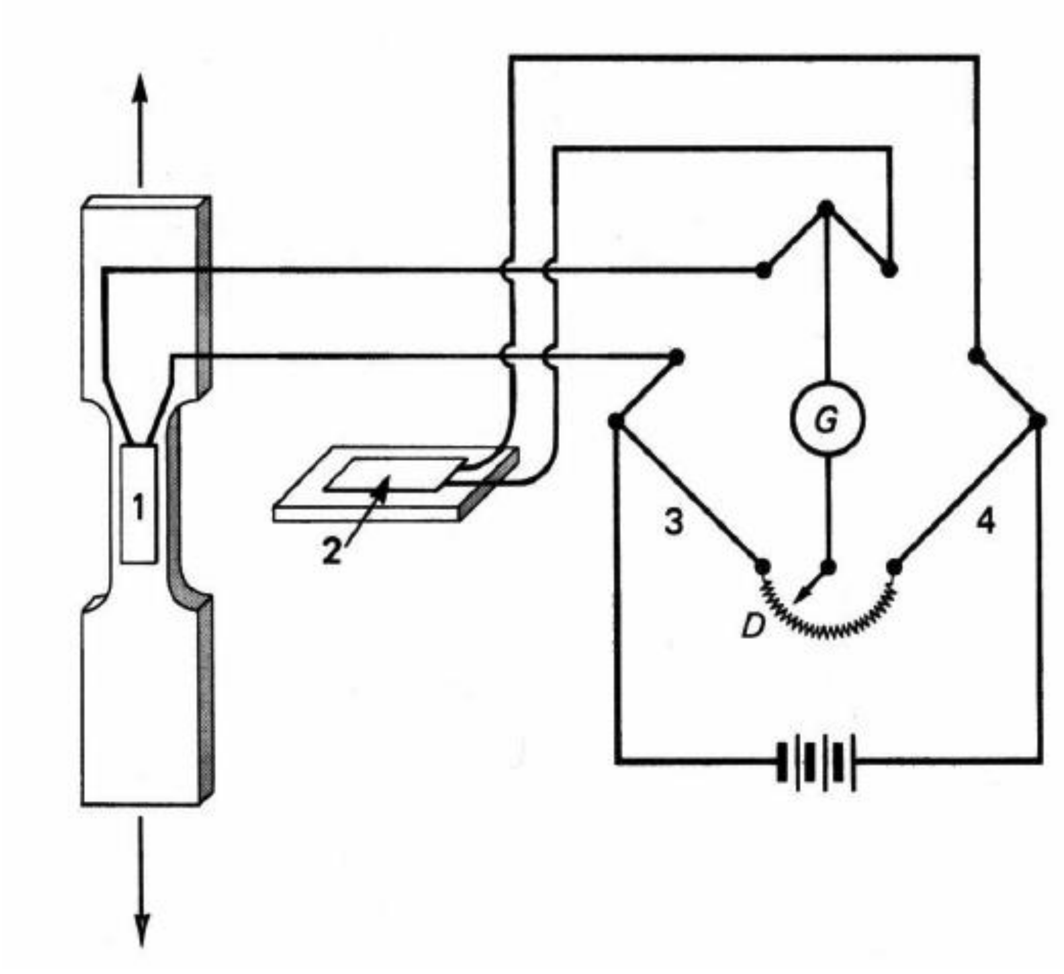


material	α_c
a titanium Ti6Al4V	$8.5 \cdot 10^{-6}/K$
b ferritic steel	$12 \cdot 10^{-6}/K$
c austenitic steel	$16 \cdot 10^{-6}/K$
d aluminum AlCuMg2	$23 \cdot 10^{-6}/K$

If $\varepsilon = \frac{1}{F}\frac{\Delta R}{R}$ how much is actually the resistance variation $\Delta R = \varepsilon \cdot F \cdot R$ if we measure a strain of *100µm/m* with a *120Ω* gage ? … $\Delta R = 100 \cdot 10^{-6} \times 2 \times 120\Omega = 0.024\Omega = 24m\Omega$

It's a variation of only the 0,02% of the base resistance ! These small variations are optimal to be measured with the ***Wheatstone Bridge …***





We have now a *combined measurement chain*:
"Strain Gage + Wheatstone Bridge"

The <u>two graduation curves</u> work then together …

$$\varepsilon = \frac{1}{F}\frac{\Delta R}{R} \qquad \frac{\Delta e}{E} = \frac{1}{4}\frac{\Delta R}{R}$$

Which combined give us: $\frac{\Delta e}{E} = \frac{1}{4} \cdot F\varepsilon$

$$\boxed{\Delta e = \frac{1}{4} EF \cdot \varepsilon}$$

the combined graduation curve !

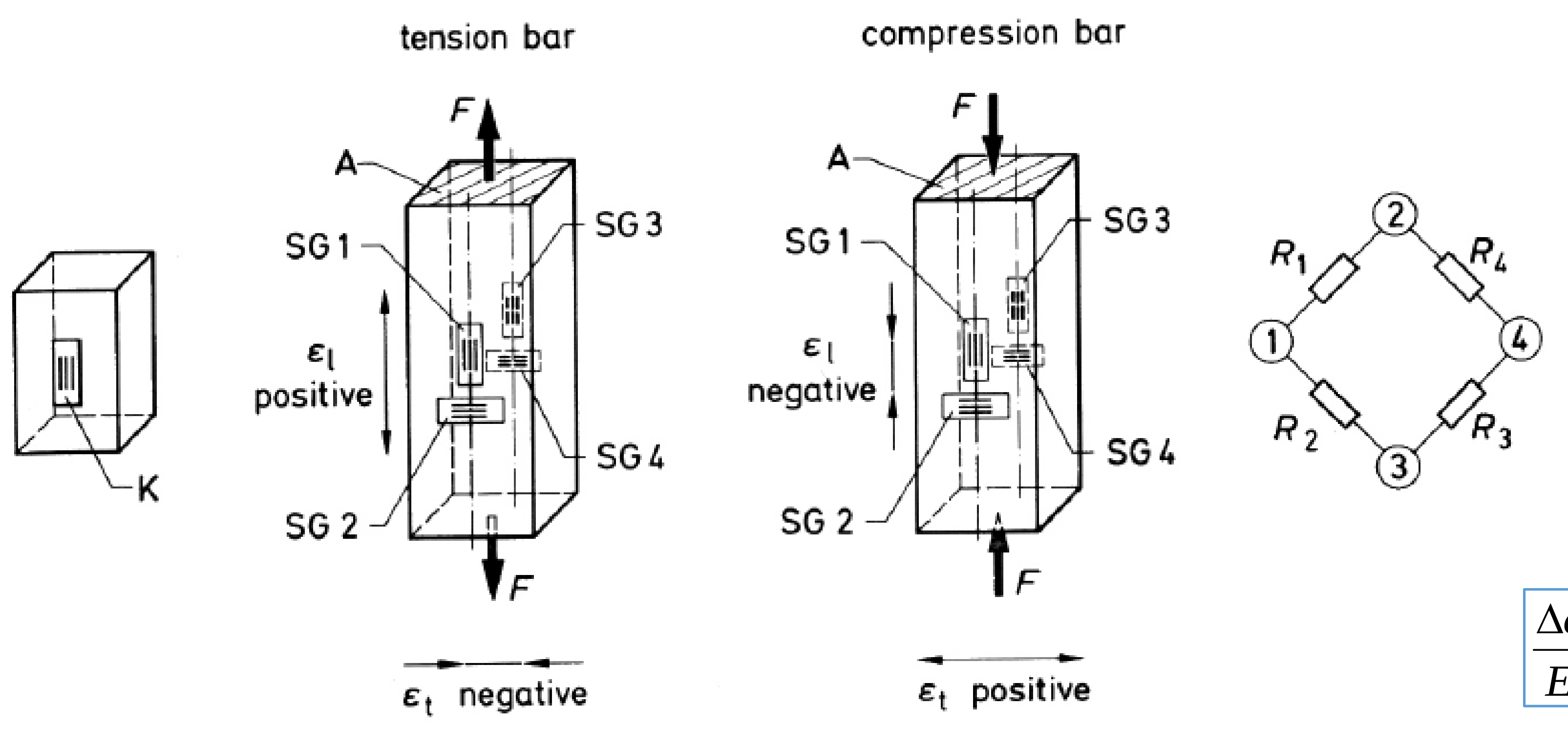
Strain gage (2) *does not measure any strain* but, it *undergoes the same thermal effects* of strain gage (1)

$$\frac{\Delta R_{T1}}{R} = \frac{\Delta R_{T2}}{R} = \left[\alpha + F\left(\beta' - \beta''\right)\right] \cdot \Delta T$$

If the two strain gages (1) and (2) are applied on <u>*contiguous arms*</u> of the Wheatstone Bridge, the thermal effects are "automatically eliminated": $\frac{\Delta e_T}{E} = \frac{1}{4}\left(\frac{\Delta R_{T1}}{R} - \frac{\Delta R_{T2}}{R}\right) = 0$

This is perhaps the *most important service* the WB does during strain measurement !

Measurement of tensile strain:

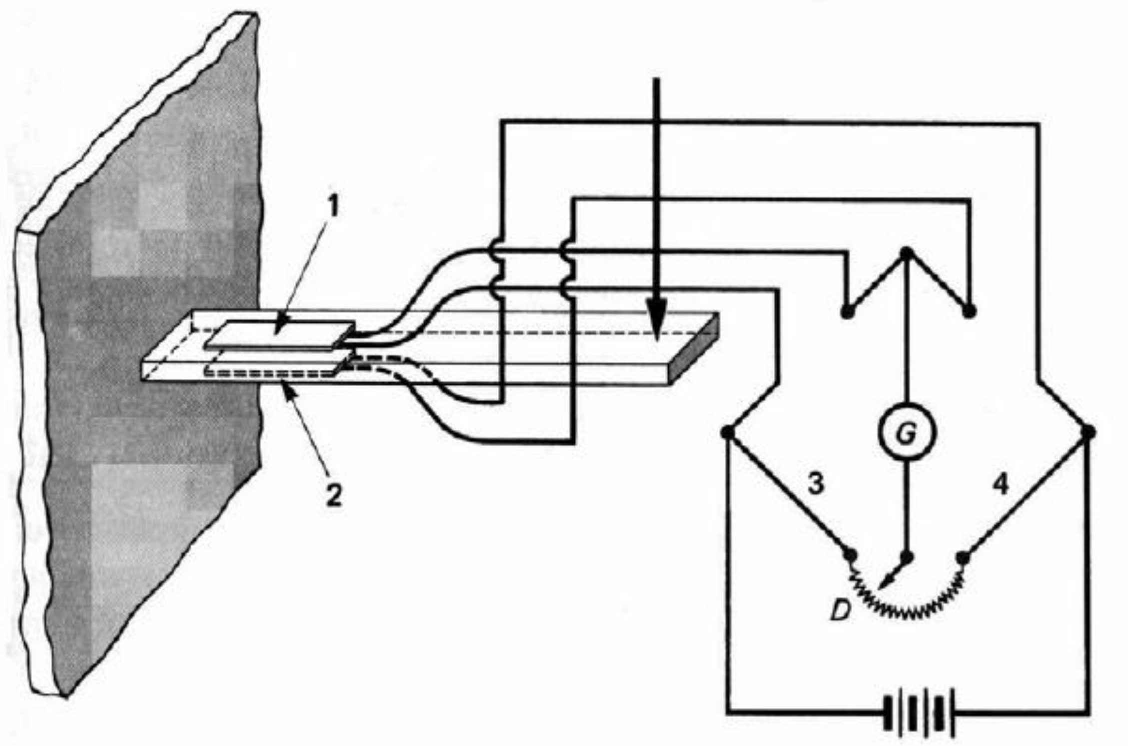


The second gage (2) can be applied more efficiently on the specimen to increase the measurement sensitivity, but it must be <u>*rotated 90° in a XY configuration*</u> to give a useful signal. It will then measure the transversal strain $\varepsilon_t = -\nu \cdot \varepsilon_a$ producing the graduation curve:

$$\frac{\Delta e}{E} = \frac{F}{4}(\varepsilon_a - \varepsilon_t) = \frac{F}{4}(\varepsilon_a - (-\nu\varepsilon_a)) = \frac{F}{4}\varepsilon_a(1+\nu)$$

where the factor ***(1 + v) = 1,3*** is an extra amplification *due only to the bridge configuration,* called ***«bridge factor»***

Measurement of bending strain:



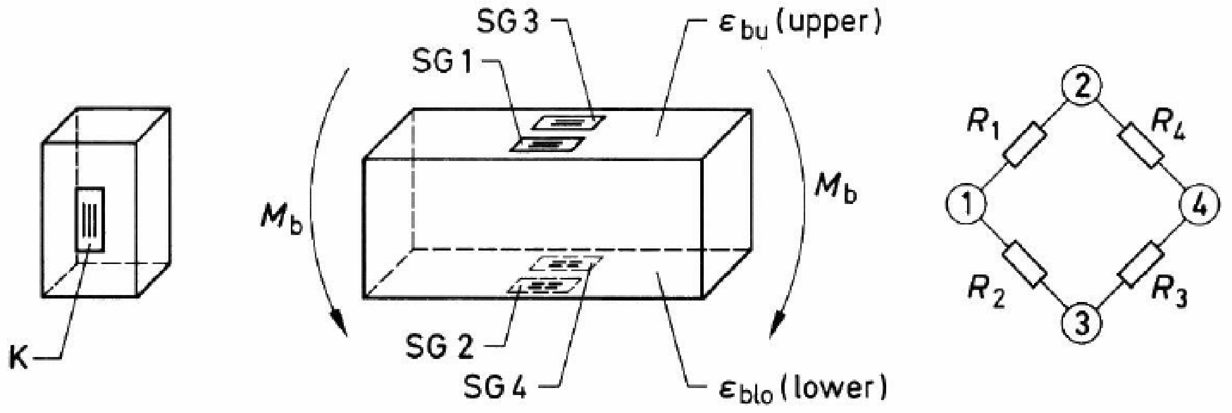
the upper gage measures the <u>*stretched fiber strain*</u> while the bottom gage measures the <u>*compressed fiber strain*</u> :

$$\frac{\Delta e}{E} = \frac{1}{4} F\left(\varepsilon_{f1} - \varepsilon_{f2}\right)$$

$$\frac{\Delta e}{E} = \frac{F}{4}\left(\varepsilon_{f1} - \left(-\varepsilon_{f1}\right)\right) = \frac{F}{4} \cdot 2\varepsilon_f = \frac{F}{2}\varepsilon_f$$

because it is $\varepsilon_{f2} = -\varepsilon_{f1}$

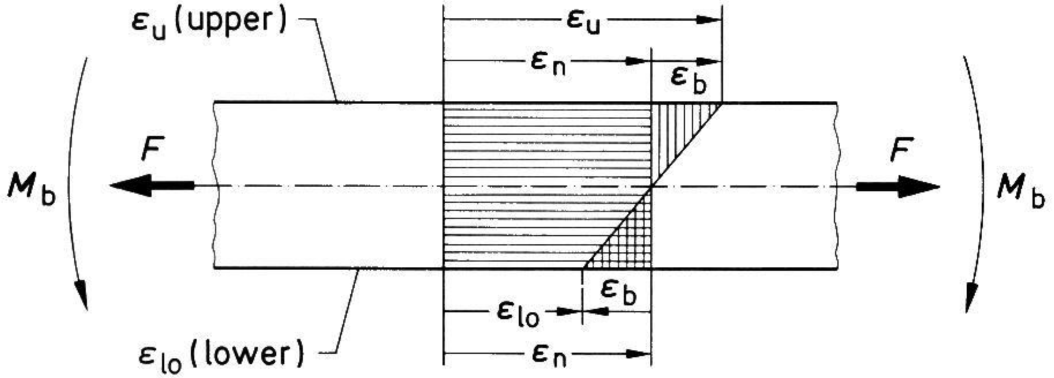
Ad the *"bridge factor"* is equal to 2 !



Note that the *bridge factor can be increased* up to 4 if we employ four strain gages connected as shown on the left and make the full bridge active !

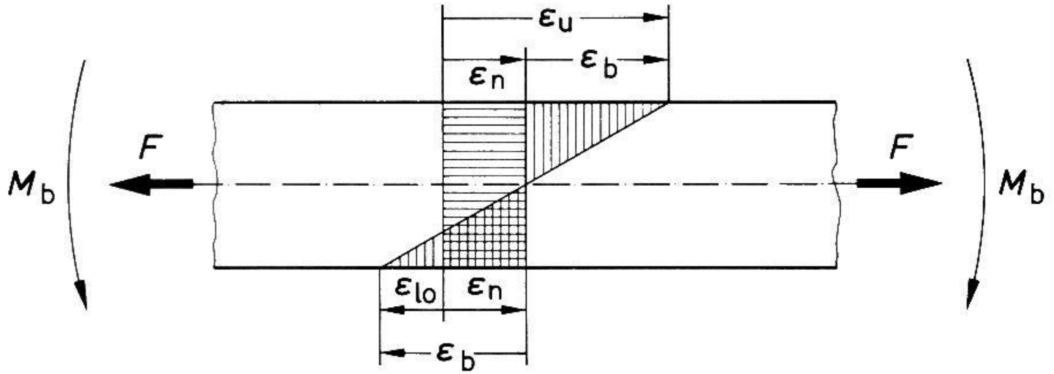
$$\varepsilon_1 = \varepsilon_3 = -\varepsilon_2 = -\varepsilon_4$$

Strains, of course, can be composed in more complicated ways , as shown in the figures ...



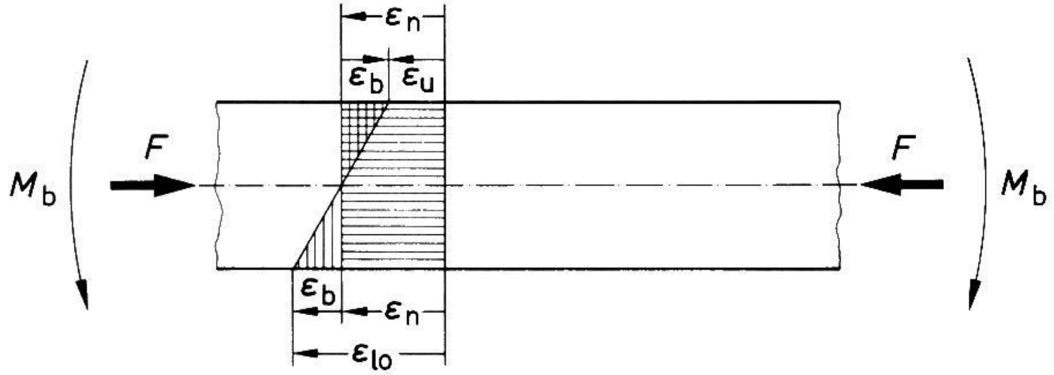
case a: $\varepsilon_n > \varepsilon_b$

ε_u and ε_{lo} are both positve



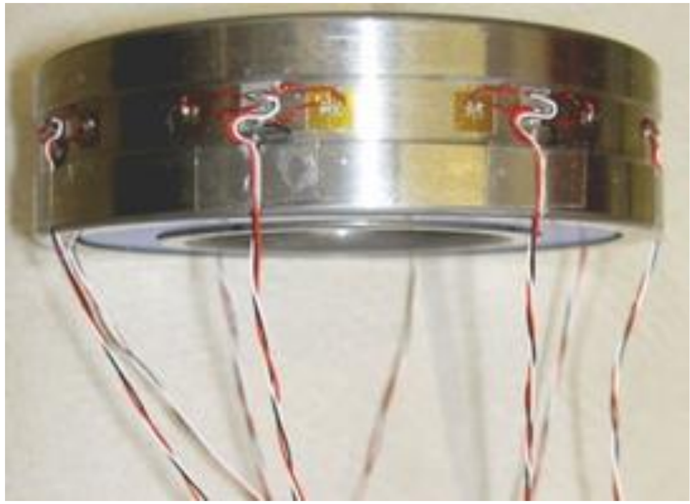
case b: $\varepsilon_n < |\varepsilon_b|$

ε_u positive,
ε_{lo} negative

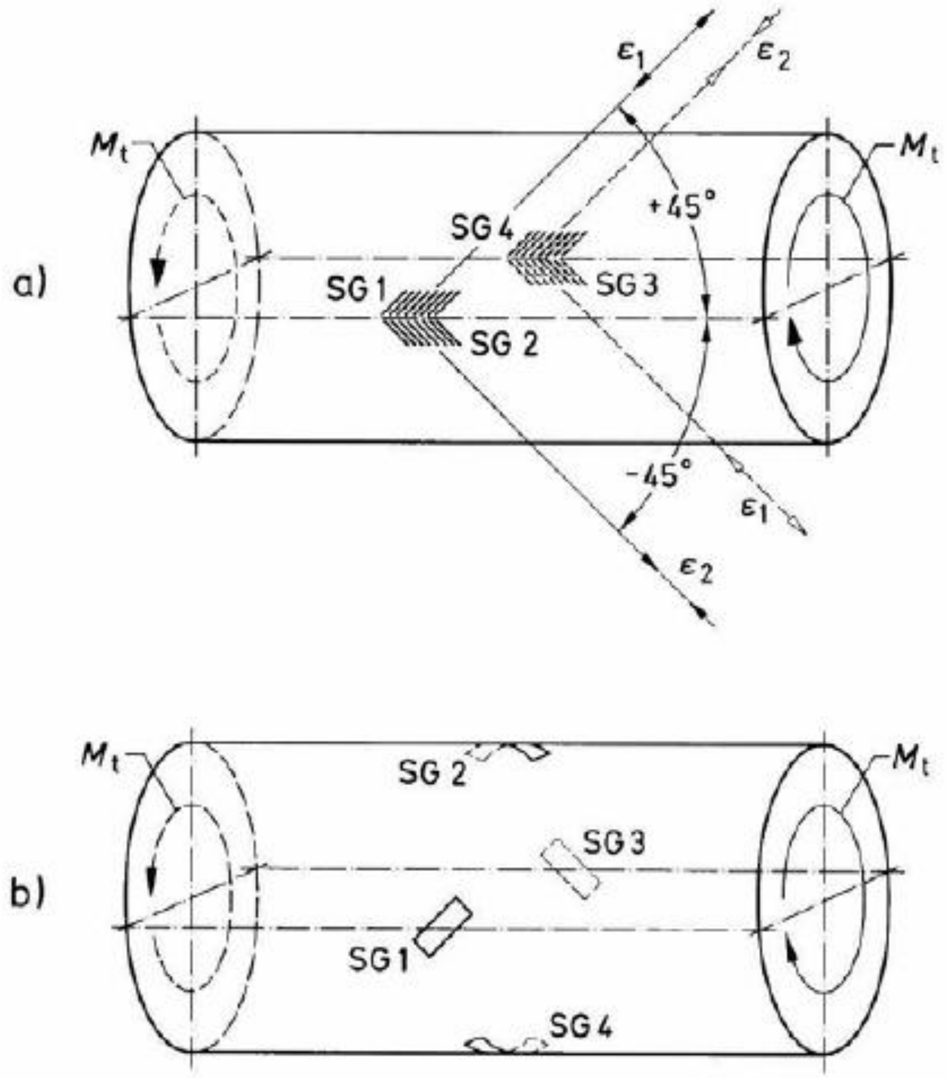


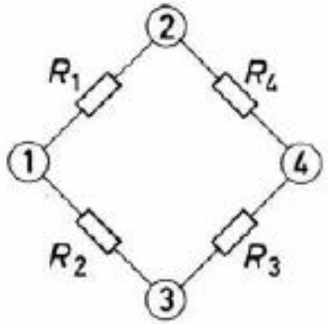
case c: $-\varepsilon_n > \varepsilon_b$

ε_u and ε_{lo} are both negative



Measurement of torsion strain:





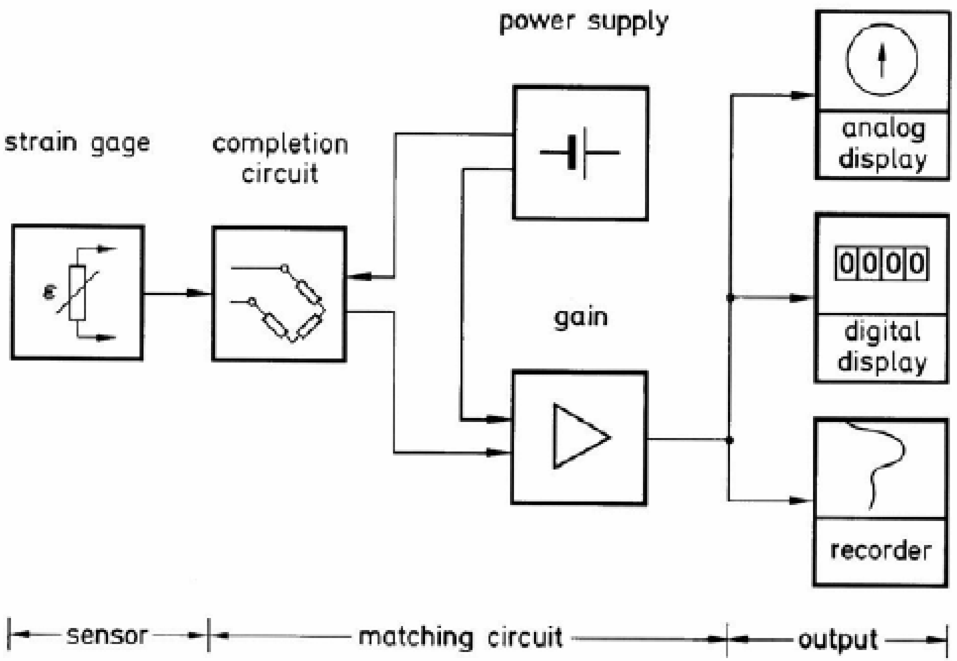
Fundamental application for the crankshafts, where we wont to measure the ***drive torque*** to get the *power output* of an engine:

$$W_{mecc} = C \times \omega$$

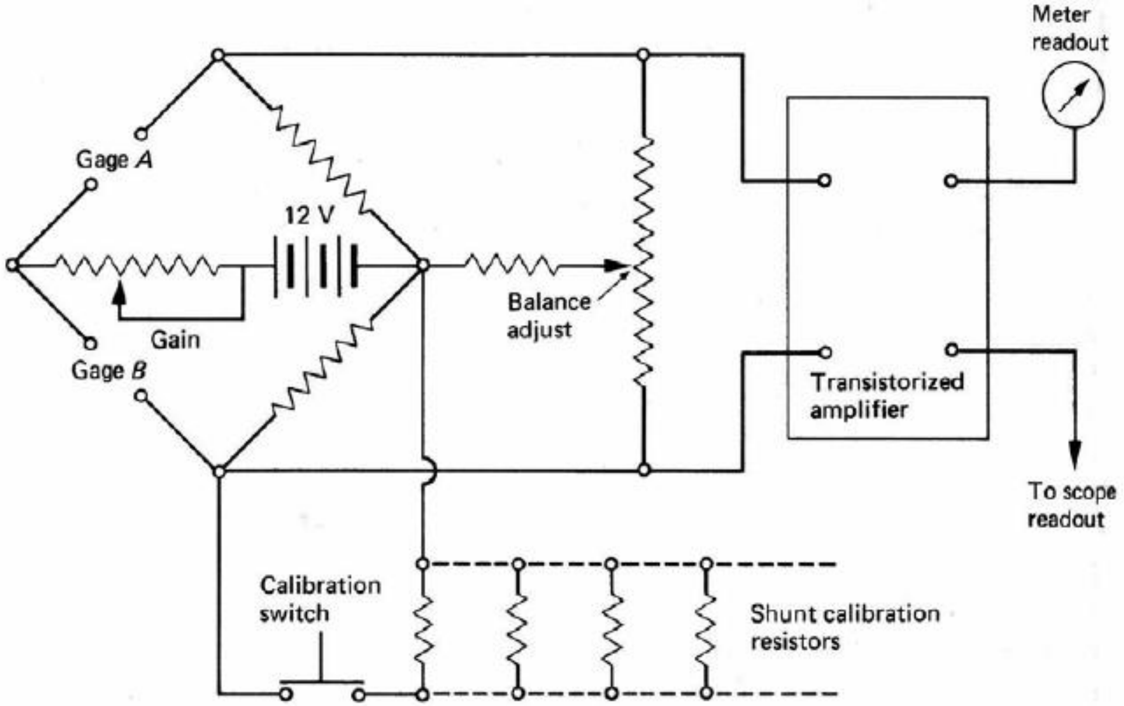
Strain gages are placed as in the figure, aligned with the maximum strain 45° lines on the shaft surface. The strains will be $\varepsilon_1 = \varepsilon_3 = -\varepsilon_2 = -\varepsilon_4$ and the graduation curve results:

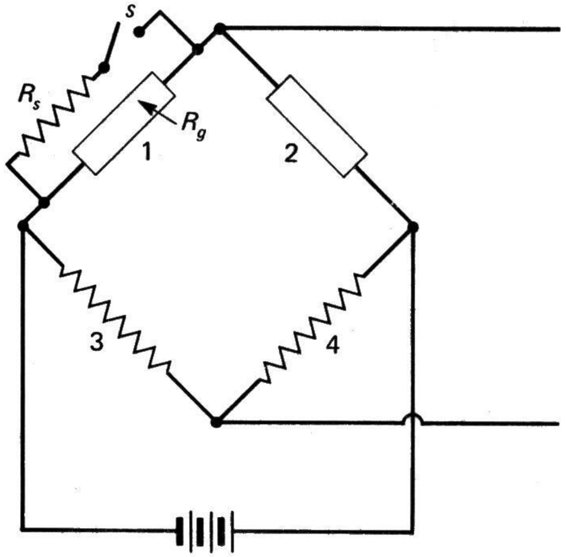
$$\frac{\Delta e}{E} = \frac{F}{4}\left(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4\right) = \frac{F}{4} \cdot 4\varepsilon_{45°}$$

with a *full bridge factor* equal to 4 !



Examples of complete strain gage measurement chains :

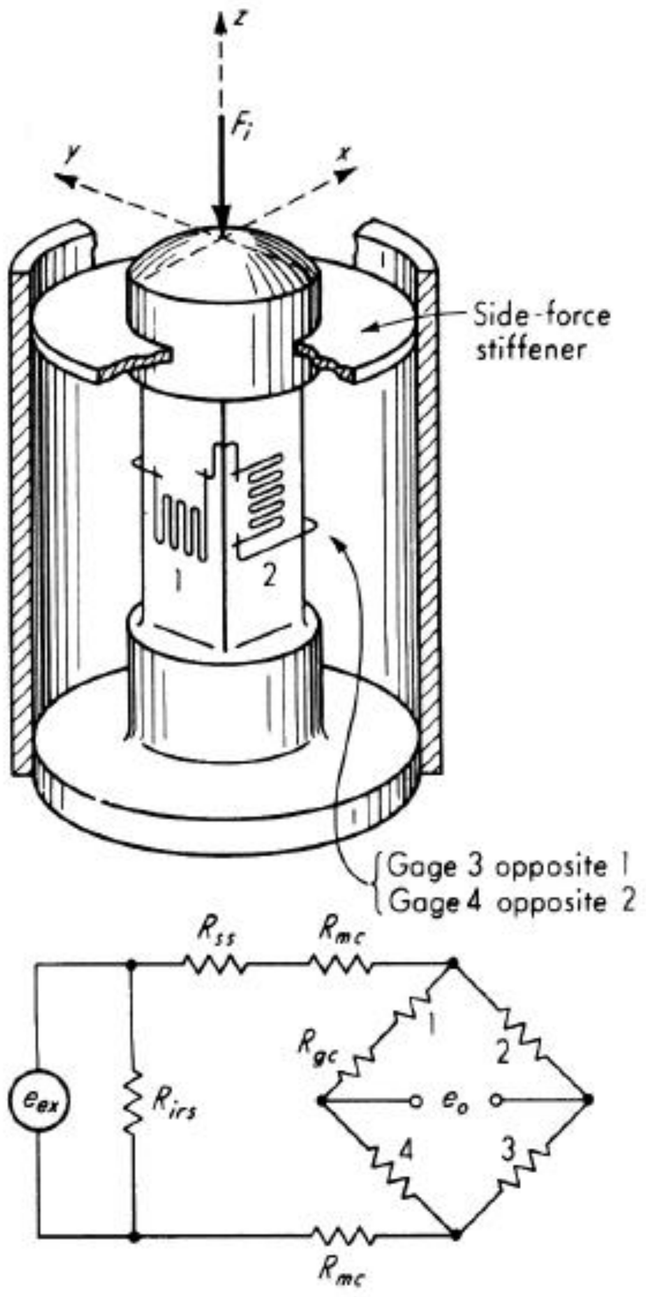




Note that strain gage channels need to be *calibrated* before use, this is done by connecting internally shunt resistors R_s that simulate specific strain values:

$$\frac{\Delta R}{R_g} = F \cdot \varepsilon_{elettrica} \quad \text{with} \quad \Delta R = R_g - \frac{R_g R_s}{R_g + R_s} \quad \text{and} \quad \frac{\Delta e}{E} = \frac{1}{4}\frac{\Delta R}{R_g} = \frac{1}{4}\left(F \cdot \varepsilon_{elettrica}\right)$$

Example of strain gage application in a Load Cell to measure a force



Physical basic equation: $$F = k \cdot x$$

where (for tension and compression) $$k = \frac{AE}{l}$$

Graduation curve is linear : $$x = \frac{1}{AE} \cdot F$$

And *sensitivity is constant* : $$S = \frac{\Delta x}{\Delta F} = \frac{1}{AE}$$